

Acoustic pressure losses in woven screen regenerators

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ABSTRACT

In thermoacoustic travelling-wave engines and other Stirling cycle devices, good performance depends on the material of a regenerator being in intimate contact with the gas inside it, so that each particle of gas oscillates in temperature following the adjacent material as it is acoustically displaced. This requires that the passages are small enough for temperature waves to penetrate across the gas path with the frequencies of interest. One type of 'regenerator' that is commonly used for this purpose is composed of multiple layers of woven stainless steel mesh, laid on top of one another in random registration. Associated with the thermal penetration is a viscous loss of pressure and this must be quantified if efficient engines are to be designed.

In the literature, reliance has been placed on the correlation of steady-flow loss data for these meshes, but for the coarser ones operating at frequencies greater than 28 Hz, the assumption of quasi steady-flow is dubious and direct acoustic measurements must be made. This paper reports acoustic pressure loss data for meshes with 34 and 75 wires per inch taken in two configurations of impedance tube, and finds that the dependence on velocity is the same as in steady-flow, but that there is indeed some enhancement of loss for frequencies above 40 Hz. (Separation of the mesh layers is probably responsible for the anomalously low loss coefficients that were recorded in one set of data.) It is shown that the acoustic pressure losses can be correlated in terms that give the acoustic impedance more directly than the friction factor correlations.

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1. Introduction

In the performance of thermoacoustic heat engines, the characteristics of the material in the regenerator are of paramount importance [1]. A temperature gradient is imposed on the regenerator by heat exchangers at each end and the function of the material is to ensure that acoustic waves passing through it are in thermal contact to some extent.

In a 'travelling wave' engine, although the wave may be far from a pure travelling one, the contact should be intimate so that all the particles of gas follow the temperature of the adjacent material, and thus undergo a Stirling cycle as the gas is displaced alternately towards the hot end (while compressed) and the cold end (while expanded). This requires that the frequency is low enough for temperature waves to penetrate across the gas path, and hence (even for moderate frequencies) that the gas passages be very small. However, this must be achieved without incurring so much viscous or thermal dissipation that the work input to the acoustic wave is negated. Depending on the acoustic conditions, both of the mechanisms may be important, but in a high impedance regenerator,

viscous dissipation tends to dominate. Thus, it is essential to know the acoustic pressure loss characteristics of the material so as to be able to quantify the viscous dissipation.

One type of 'regenerator' that is commonly used in travelling wave devices is composed of multiple layers of woven stainless steel mesh, laid on top of one another in random orientation. This note describes measurements of the acoustic characteristics of a 10 mm thickness of two different mesh layers, and compares the performance with the correlation presented by Swift and Ward [2]. The correlation was derived from the steady-flow data quoted by Kays and London [3] from the experiments of Tong and London [4]. It has proved its relevance to thermoacoustic conditions through incorporation in engine calculations. However, with an acoustic wave there must be differential acceleration of different regions of the gas at the microscopic level within the mesh, so it would not be expected that the steady-flow data can be applied precisely. Direct measurements of the acoustic losses are presented here, thus contributing to the filling of a significant gap in the literature.

2. Pressure loss coefficient measurements

Schematic diagrams of two realizations of the apparatus are shown in Fig. 1. They were both excited by two Pioneer sub-woofer

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Nomenclature

A	face area of regenerator	k	thermal conductivity
C_p	thermal capacity	p	acoustic pressure, or pitch of woven wire mesh
G	peak volumetric flow rate at regenerator mid-point	p_a	acoustic pressure
K''	dimensional loss coefficient	r_h	hydraulic radius
L	regenerator depth	t	time, or thickness of wire in the mesh
L_{eff}	effective depth of regenerator including end-corrections for gas inertia	u', u	local acoustic velocity, averaged across the passage
LC	dimensionless pressure loss coefficient	u_{pore}	notional acoustic velocity in the pores of the mesh, based on porosity, at the mid-point of the regenerator
P	mean pressure	x	axial distance
Pr	Prandtl number	y_0	channel half-width
Re	Reynolds number based on fixed length	y, z	transverse distances
T	mean temperature	α	thermal diffusivity
T'_a, T_a	acoustic temperature, averaged across the passage	γ	ratio of specific heat capacities
U_{pore}	mean velocity in the pores of the mesh, based on porosity	δ_v, δ_k	viscous, thermal penetration distances
V_{pk}	peak voltage driving the sub-woofer amplifiers	ζ	specific acoustic impedance $\equiv z/\rho c$
Z	acoustic impedance (p/u)	ϑ	parameter in loss coefficient correlation
c	speed of sound	$\vartheta_{scale}, \vartheta_{vel}$	parameters in proposed variation of ϑ
$c_1(\varphi), c_2(\varphi)$	parameters in the correlation of Swift and Ward [2]	μ	dynamic viscosity
d_e	equivalent diameter of the gas passage	ν	kinematic viscosity
f	Fanning friction factor, defined by Eq. (1)	ρ	mean density of gas
f_v	Rott's acoustic parameter obtained by integrating h_v across the passage	ρ'_a, ρ_a	acoustic fluctuation in the density of the gas, averaged across the passage
f_k	Rott's acoustic parameter obtained by integrating h_k across the passage	τ_{diff}	time constant of diffusion
$h_v(y, z)$	function describing the variation of the acoustic velocity field	ϕ	porosity
$h_k(y, z)$	function describing the variation of the acoustic temperature field	ω	circular frequency

Subscripts

1–7 position of microphones in Fig. 1

loudspeakers, rated at 1 kW each, mounted in an MDF enclosure so that the majority of the sound emanated from a 250 mm diameter orifice in the side of the enclosure. In the first series of experiments, the sound then propagated through 530 mm of a horizontal 300 mm diameter steel duct, halfway along which was mounted the mesh assembly, before exhausting to the laboratory. The assembly was a 10 mm sandwich of layers of plain weave mesh between two supporting layers of coarse grid. These coarse grids were required to prevent the layers of mesh under test from sagging or springing apart, and they were assumed to make a negligible contribution to the pressure loss. In the second series of experiments, suggested by one of the referees of the first draft of this paper, the duct was extended by 200 mm and the sandwiches of mesh were installed 20 mm from the open end. A second 10 mm sandwich of coarse mesh was placed near the orifice of the enclosure, to enhance the radial uniformity of the acoustic field.

2.1. Technique for the first series

The positions of four 6 mm holes in the 2 mm thick stainless steel pipe are also shown in Fig. 1a. The tube mountings communicating with these holes were designed to support 13 mm B&K microphones. The calibration for these microphones was obtained before each experiment with a 1 kHz source giving 10 Pa, and assumed to apply at the lower frequencies of the experiment, on the basis of the manufacturers' data.

In the first series of experiments, driven by the need for high accuracy in the phases in particular, pressure ratios and phase relationships were recorded using the same microphone. To obtain the phase relationships between positions, the microphone signals were each referenced to the current of the sub-woofer driving signal. Using the ratios of magnitude and differences of phase, the

two-microphone method [5] was then applied to deduce the acoustic velocities. Thus, the velocities at stations 1 and 2 for example were given by

$$u_1 = \frac{-p_2 + p_1 \cos\left(\frac{\omega x}{c}\right)}{i\rho c \sin\left(\frac{\omega x}{c}\right)},$$

and

$$u_2 = -i \frac{\sin\left(\frac{\omega x}{c}\right)}{\rho c} p_1 + \cos\left(\frac{\omega x}{c}\right) u_1,$$

where x is the separation distance, ω the circular frequency, c the speed of sound, and ρ the density of air at ambient conditions.

Pressures and velocities at stations 1 and 2, and also 4 and 5, were extrapolated to the centre of the mesh, also using the pipe transmission equations. From these data, the pressure loss Δp and the velocity in the middle of the mesh, before its correction for the porosity to give u_{pore} , were deduced. The velocities deduced by extrapolation from stations 1 and 2 were 5–20% greater than those from 4 and 5. The value from the loudspeaker side was taken to be representative in working out the loss coefficient because the larger phase differences there should yield higher accuracy.

In collecting the data, the voltage generated by the amplifier driving the sub-woofers was fixed. The acoustic wave incident on the regenerator was varied by varying the frequency from 112 Hz down to 28 Hz, the range of interest in the present thermoacoustic studies. A second and higher voltage was then used to provide overlapping and larger values on the velocity scale. Electrical powers up to 500 W were employed, but the maximum transmitted acoustic power in the duct was only about 1 W, because a predominantly standing wave was generated. Typical deduced profiles of acoustic pressure and velocity are shown in Fig. 2.

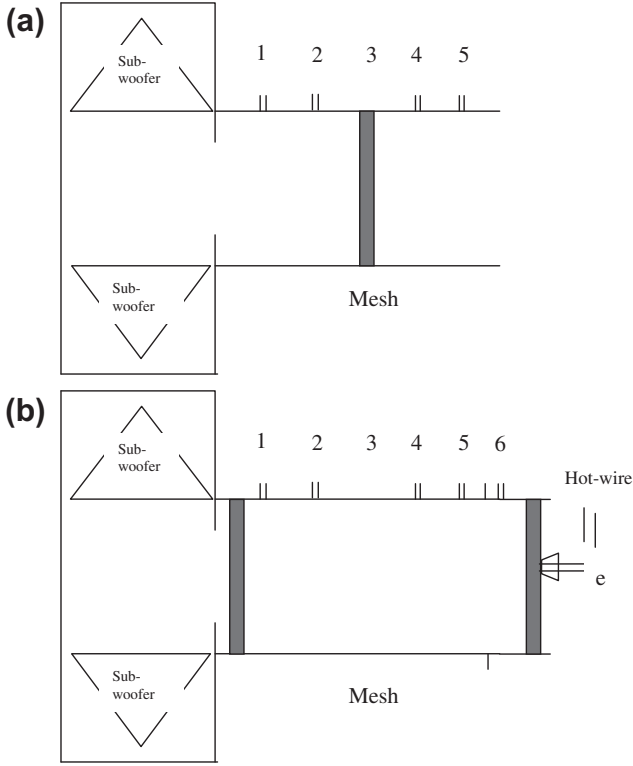


Fig. 1. (a) First configuration of the experiment with four microphone positions. (b) Second configuration with three active microphone positions (4, 5 and 6).

2.2. Technique for the second series

In the second set of experiments, because of the absence of sufficient duct between the meshes and the open end, it was necessary to measure the acoustic pressure at the outlet by pressing a 13 mm rubber bung containing a 6 mm GRAS microphone against the supporting grid, as shown in Fig. 1b. The same design of rubber mounting supported two similar microphones in two of the three tubes at the positions shown. After checks that interchanging the microphones, with their slightly different phase characteristics,

did not affect the results substantially, it was decided to leave one microphone at position 4 and to move the second to positions 5, 6 and the duct exit, recording the phase differences from the reference microphone and normalizing the amplitude to that of the reference. This was necessary because of small variations in amplitude as the sub-woofers warmed up. Microphones 4 and 5 were analysed to deduce velocities and to extrapolate to the face of the mesh. Microphones 4 and 6 provided a check on this calculation. A hot-wire anemometer probe was also installed at the exit to give an approximate indication of acoustic velocity there at the time when the amplitude of excitation was being set.

3. Analysis of results

The change in the real component of the acoustic pressure relative to the velocity is a measure of the dissipative loss due to viscosity (see the Appendix A). There are also some capacitive and inductive effects, but these are small in comparison. In addition there is dissipation due to thermal effects giving rise to a change in the acoustic velocity. However, it is shown in the Appendix A that the ratio of the viscous to thermal dissipation is very large in these experiments, and that the change in velocity can be neglected.

Anticipating the use of the data in thermoacoustic network calculations, an acoustic pressure loss coefficient is defined here as $LC \equiv \text{real} \left\{ \frac{\Delta p}{\rho c U_{\text{pore}}} \right\}$, the change in specific impedance, where U_{pore} is the amplitude of velocity at the regenerator mid-point. This is calculated from the peak volumetric flow rate at that point G and the '3-D porosity' ϕ as $U_{\text{pore}} = \frac{G}{\phi A}$, where A is the face area of the mesh.

To a good approximation, $\phi \equiv 1 - \frac{\pi t}{4p} \sqrt{1 + \left(\frac{t}{p}\right)^2}$ for a mesh with wire diameter t and pitch p . However, the square root term which expresses the additional length of wire due to its tortuosity in the woven form is sometimes neglected.

Of substantial interest is the relationship of the pressure losses in acoustic flows to those under steady-flow conditions. The steady-flow loss data reported by Kays and London [3] have been coded by Swift and Ward [2] in the form of correlations of the Fanning friction factor with mean velocity U_{pore} as

$$f \equiv \frac{2\Delta P r_h}{\rho U_{\text{pore}}^2 L} = \frac{c_1(\phi)}{\text{Re}} + c_2(\phi), \quad (1)$$

where the hydraulic radius is $r_h \equiv \frac{\text{gas volume}}{\text{surface area}} = \frac{p}{\pi} - \frac{t}{4}$ and is $1/4$ of the equivalent diameter de . L is the regenerator thickness and the Reynolds number $\text{Re} = \frac{\rho de U_{\text{pore}}}{\mu}$. The first term in this equation expresses the viscous component of the pressure loss, and the second, the separated flow component, as illustrated by the manipulation of (1) to give

$$\Delta P = \frac{L}{de} \frac{\mu U_{\text{pore}}}{2r_h} c_1(\phi) + \frac{4L}{de} \frac{\rho U_{\text{pore}}^2}{2} c_2(\phi). \quad (2)$$

Swift and Ward [2] propose $c_1(\phi) = 1268 - 3545\phi + 2544\phi^2$ and $c_2(\phi) = -2.82 + 10.7\phi - 8.6\phi^2$. Defining

$$K'' \equiv \frac{2fL\rho U_{\text{pore}}}{de} = \frac{\Delta P}{U_{\text{pore}}}, \quad (3)$$

the first and second terms in K'' are typically of comparable magnitude for our meshes when $U_{\text{pore}} = 3$ m/s. For acoustic waves, Swift and Ward average the square term in Eq. (2) across the flow to obtain

$$K'' \equiv \text{real} \left\{ \frac{\Delta p}{U_{\text{pore}}} \right\} = \frac{\mu L}{8r_h^2} c_1(\phi) + \frac{\mu L}{r_h^2} \frac{1}{3\pi} \frac{\rho |U_{\text{pore}}| de}{\mu} c_2(\phi). \quad (4)$$

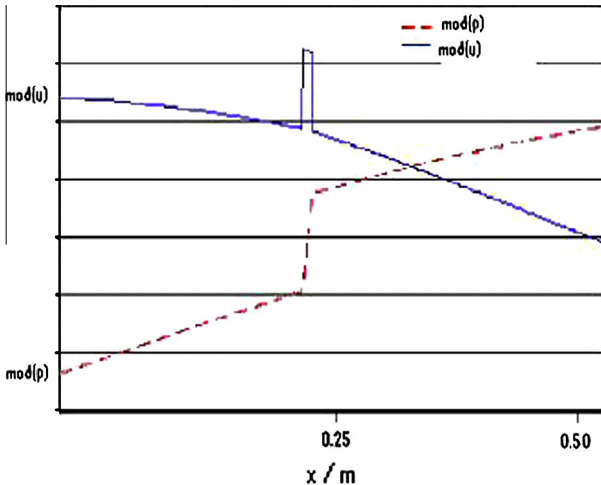


Fig. 2. Calculated profiles of acoustic velocity and pressure along the duct from the open end towards the sub-woofers, showing the acceleration of the flow and the pressure loss through the confines of the mesh, at 80 Hz.

A dimensionless representation of this loss coefficient is

$$LC \equiv \frac{K''}{\rho c}, \quad (5)$$

where c is the speed of sound and, in non-isothermal situations, both this property and the gas density are evaluated at the mid-point of the regenerator. According to (4) and (5), LC is proportional to the length of the regenerator, and is a function of the Reynolds number and the acoustic Mach number, and of the porosity. The change in the specific impedance of a duct containing a transverse regenerator (referred to the free-stream velocity) can therefore be written

$$\Delta \zeta = \frac{LC}{\phi} + j \frac{\omega L_{eff}}{\phi c}, \quad (6)$$

where the effective length L_{eff} is also deduced empirically from the imaginary component of the pressure loss relative to the velocity.

4. Results

The original choice of mesh ('coarse') for the present project was found in thermoacoustic experiments to have far from perfect thermal contact with the ~ 80 Hz waves of particular interest. A finer one was therefore also selected, with some misgivings about the obviously high resistance to flow.

The characteristic parameters of the two meshes are given in Table 1, together with three others on which steady flow tests have been conducted recently by Yu et al. [6].

In the first series of experiments, the number of mesh layers was calculated on the basis of twice the wire diameter per layer to give a 10 mm thickness: 20 for the coarse mesh and 53 for the fine one. In the case of the coarse mesh, a test was also conducted with half the number of layers, to check the linearity of the data with respect to length. In the second series, 20 layers of the coarse mesh and 60 layers of the fine mesh were stacked. These gave a thickness of 9.5 mm in the centre under light compression, in both cases, against notional values of 10.2 mm and 11.4 mm, respectively, so apparently some interleaving of the wires in adjacent layers is possible. The 12% smaller number of fine mesh layers in the same thickness of stack in the first series suggests that there was some springing apart.

The results for the first series, in which a set sub-woofer voltage was maintained for the range of frequencies, are presented in Fig. 3, with a sample given in Table 1.

The results for the second series are presented in Figs. 4 and 5 for the coarse and fine meshes, respectively. In this case, the ampli-

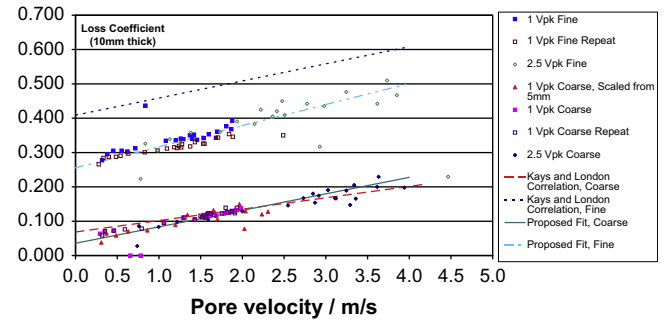


Fig. 3. Variation of the loss coefficient with velocity through the pores of the mesh for the first series. Comparison with values derived from the friction factor correlation of Swift and Ward [2] and the fit proposed in Section 5. The V_{pk} values refer to the voltage driving the amplifiers for the sub-woofers. The frequency range was 28–112 Hz.

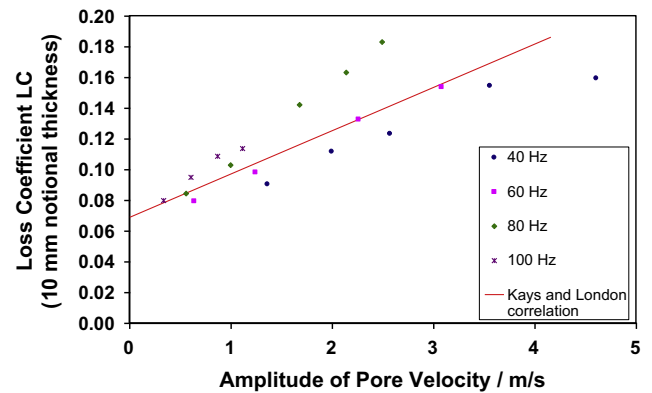


Fig. 4. Variation of the pressure loss coefficient with the amplitude of the velocity in the pores of the coarse mesh and the frequency of excitation in the second series. Comparison with the steady-flow correlation of Kays and London [3], as interpreted by Swift and Ward [2].

tude of the acoustic velocity through the mesh at a fixed frequency was maintained approximately at a number of levels by adjusting the sub-woofer voltage while observing the rms hot-wire signal.

The results are presented in terms of the pore velocity, rather than the Reynolds number $Re = \frac{4u_{pore}r_h}{\nu}$, because Eqs. (4) and (5) show that the second term is actually dependent on the acoustic Mach number, $\frac{u_{pore}}{c}$. However, this term reflects the influence of the 'form drag' of the wires in the mesh. The Reynolds number

Table 1

Dimensions of meshes investigated here and by Yu et al. [6]. Results for loss coefficient from acoustic experiments and for steady flow, with a mean pore velocity of 3 m/s, which gives an acoustic displacement at 80 Hz of 6 mm. Comparison with the correlation derived by Swift and Ward [2] from the steady flow data of Tong and London [4] reported by Kays and London [3].

Source	'Coarse' mesh of Yu et al.	'Coarse' mesh	'Medium' mesh of Yu et al.	'Fine' mesh	'Fine' mesh of Yu et al.
Wire diameter (mm)	0.280	0.254	0.230	0.095	0.089
Mesh #	30	34	45	75	94
Wire pitch (mm)	0.847	0.747	0.564	0.340	0.270
3-D porosity	0.727	0.733	0.654	0.781	0.728
2-D porosity	0.448	0.436	0.351	0.519	0.450
Hydraulic radius r_h (mm)	0.200	0.174	0.122	0.084	0.064
Length (mm)	50	10 (or 5)	50	10	50
Frequency (Hz)	20–120	28–112	20–120	28–112	20–120
With a pore velocity of 3 m/s and 10 mm regenerator thickness at 80 Hz					
Reynolds number	151	132	92	64	48
Acoustic LC scaled to 10 mm	0.01	0.19 (1st)	0.03	0.44 (1st)	0.14
		0.21 (2nd)		0.66 (2nd)	
Yu et al. measurements giving LC in steady flow	0.13	–	0.21	–	0.63
Swift and Ward correlation giving LC	0.14	0.17	0.32	0.56	0.78

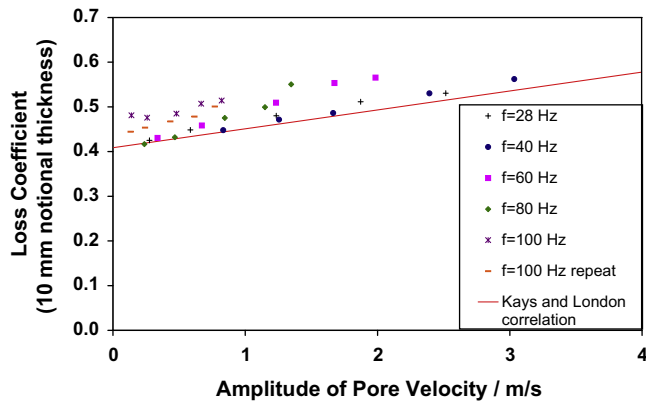


Fig. 5. Dependence of the pressure loss coefficient with the amplitude of the velocity in the pores of the fine mesh and the frequency of excitation in the second series. Comparison with the steady-flow correlation of Kays and London [3], as interpreted by Swift and Ward [2].

ranged up to 60 for the fine mesh, and up to 200 for the coarse, and the Mach number up to 0.009 and 0.013, respectively. What clearly emerges from Figs. 4 and 5 above is that there is also a dependence on the frequency. This can be expressed in dimensionless terms as dependence on $\left(\frac{r_h}{\delta_p}\right)^2 = \frac{\omega r_h^2}{2\nu}$.

5. Discussion

5.1. Consistency of the results

In the first series of results, satisfactory agreement in the scaling from 5 mm to 10 mm for the coarse mesh is shown, and also repeatability to within 2% for both meshes. However, the range of amplitude of the pore velocity varied with frequency. Because of the characteristics of the sub-woofers and enclosure, the magnitude of the velocity rose to peaks at 40 and 80 Hz and then fell rapidly with increasing frequency, so the associated acoustic displacement with the coarse mesh and 1 V_{pk} excitation peaked at 40 Hz at about 8 mm, and fell very rapidly above 80 Hz, where it was 4 mm. Corresponding figures for 2.5 V_{pk} excitation were 14 mm and 7 mm, respectively, so end effects due to some of the interacting air particles spending significant time outside the mesh might have affected the data for this case, but the agreement of the data for 5 mm and 10 mm lengths suggests otherwise.

There remained the possibility that an effect of frequency was superimposed upon that of velocity. However, since the results for different intensities of excitation (1 and 2.5 V_{pk} in the sub-woofer driving signal) agreed adequately at the same velocity, but different frequency, this possibility seemed to be negated. There was a wide enough range of data to establish a closely linear dependence of the loss coefficient on the velocity that was independent of frequency.

However, when the second series of experiments was undertaken, one objective was to look systematically for any influence of frequency. This emerged in the data for both the coarse and the fine meshes. The results for all frequencies, apart from 100 Hz with the fine mesh (Figs. 3 and 4), agreed closely with the Kays and London correlation [3] for steady flow at low acoustic velocities. However, as the amplitude increased, the higher frequency loss coefficients became greater than the steady-flow ones. Unfortunately, in the case of the coarse mesh, the results for 40 Hz fell below the steady correlation, so that when the enhancement in loss coefficient over the correlated values is plotted against the dimensionless frequency $\left(\frac{r_h}{\delta_p}\right)^2$ (Fig. 6), the two meshes show similar variations individually but an inconsistent variation when

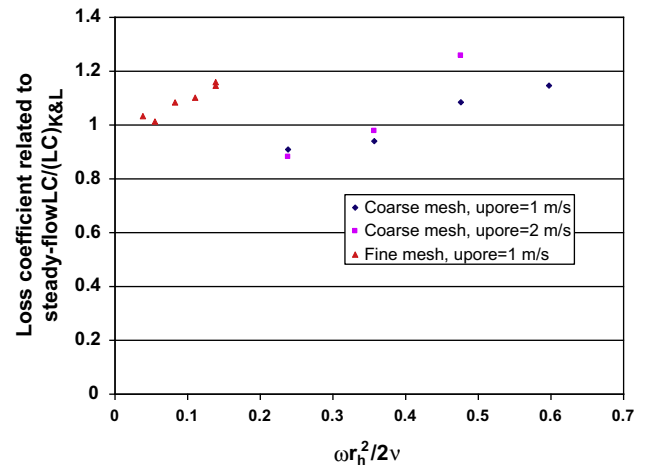


Fig. 6. Variation of the loss coefficient normalized by the steady-flow value of Kays and London [3] with dimensionless frequency for two meshes and two velocities.

combined. Because the penetration depth is becoming comparable to the hydraulic radius, a greater enhancement would be expected in the coarse mesh case. An increase in the coarse mesh loss coefficients by about 30% relative to the steady-flow values would be required to fully rationalize the data.

The other objective of the second series was to check that the thermal dissipation, which was reduced at the new location of the mesh in the duct, was indeed negligible. Substitution of the measured impedances on the mesh faces into the expression (A20) in the Appendix A suggested that the changes in acoustic velocity were insignificant compared with those in the pressure in the first series, and even smaller in the second series.

The correspondence of the first and second series of data for the coarse mesh (two assemblies, possibly of slightly different thickness) was to within the estimated precision of measurement (see Section 5.2), given the range of frequencies superimposed in the first set. However, there is a clear discrepancy between first and second sets in the fine mesh results, the nominally 12% thicker second stack giving losses some 30% greater than the first and agreeing closely with the steady-flow correlation at low frequency. Other tests have indicated that this is most likely explained by the poor compaction of the stack in the first case.

5.2. Assessment of accuracy

Clearly, adjacent pairs of holes are closer (0.33 pipe diameters) than is desirable for the two-microphone method because of the requirement for very accurate phase measurement. However, the shortest wavelengths were at least ten times this diameter so the wave should have been planar to a high degree. As a check on the accuracy of the data, a sensitivity analysis was carried out to assess the possible effect of systematic errors in the pressures on LC, particularly in recognition of this very short distance between the pressure measurement locations.

Comparison of the magnitude of the velocities deduced for position 4 in the first series from the pairs of microphones on the two sides of the mesh showed discrepancies ranging from 1% to 12%. Conceivable errors in the pressure magnitudes could not account for the discrepancies in velocities, which were predominantly in the real components. However, they could have been accounted for by an error of -0.8° in the phase of either p_2 or p_4 . A systematic measurement error of this magnitude is possible, for instance because of the positioning of the microphones relative to acoustic waves that were not truly planar at all locations.

Systematic errors in the pressure magnitudes at particular locations would have had a significant effect on the deduced values of

loss coefficient. Typically, a 1% decrease in $|p_1|$ or a 1% increase in $|p_2|$ gave rise to a 7% increase in LC , while changes in $|p_4|$ had much less influence. The possible error of -0.8° in the phase of p_4 deduced above would have given rise to the measured LC being 10–20% too low. Repeatability suggests random errors of up to 0.4° in the phases, and the resulting 10% error in LC is consistent with the scatter in Fig. 3.

In the second series for the coarse mesh, velocities and pressures at the mesh location extrapolated from the microphone pairs, $|p_4|$ and $|p_5|$, and $|p_4|$ and $|p_6|$, were within 10% of one another at 40 Hz and 60 Hz, but often differed by more at the higher frequencies. However, the boundary layer perturbation at the join between the two duct sections may have affected the accuracy of the data based on $|p_4|$ and $|p_6|$, so they were not used in the final analysis.

These are clearly only indications of the sensitivity of the results for LC , and the systematic uncertainties in the measured parameters cannot be precisely established. Overall, systematic errors of $\pm 10\%$ must be allowed and this goes some way to explaining the inconsistency in the coarse and fine mesh results in Fig. 6.

5.3. Comparison with other data

Wakeland and Keolian [7] investigated the friction losses of a wide variety of single-mesh screens and showed that f was well correlated by a function of the 'orthogonal' or '2-D projected' porosity and by a Reynolds number based on r_h . However, their maximum frequency was only 5 Hz, so $\omega\tau_{diff} \ll 1$, where $\tau_{diff} \equiv \frac{r_h^2}{\nu}$, the characteristic time for viscous diffusion. Since this is essentially the same criterion as $\frac{\omega r_h^2}{2\nu} \ll 1$, quasi-steady behaviour would be expected. This is also true of the oscillating flow data for stacked screens obtained by Tanaka et al. [8] (up to 10 Hz) and by Hsu et al. [9] (up to 4 Hz). The latter did indeed deduce that steady flow correlations applied.

At higher frequencies, some recent data for steady and acoustic flow through meshes are given by Yu et al. [6]. Results read directly from their graphs (with very limited extrapolation and not relying on their correlations) are compared with the present acoustic results in Table 1 for a pore velocity of 3 m/s. Their data are for 30#, 45#, 94#, 180# and 200# meshes. The first three are designated 'coarse', 'medium' and 'fine' to correspond approximately with the two meshes investigated in the present experiments. Results for the two even finer meshes are not included in the table.

The steady flow friction factor data of Yu et al. [6] are somewhat lower than the Swift and Ward [2] correlation if values for porosity are calculated from the published wire dimensions using the expression in Section 3. This is particularly true of their 'medium' and 'fine' meshes, which are lower by 35% and 20%, respectively. However, the acoustic data in [6] are lower by almost an order of magnitude, so these data are certainly not consistent with the present results.

5.4. Alternative formulation of viscous loss

From the analysis in the Appendix A, it may be postulated that a reasonable second-order representation of f_v for the tortuous paths represented by randomly registered meshes is

$$f_v = 1 - j \left(\frac{r_h}{\delta_v} \right)^2 \theta, \quad (7)$$

where θ is an empirical function of the Reynolds number, and possibly the non-dimensional frequency, $\left(\frac{r_h}{\delta_v} \right)^2 = \frac{\omega r_h^2}{2\nu}$ for each type of mesh. It is seen that

$$LC = \frac{1}{\theta} \left(\frac{\delta_v}{r_h} \right)^2 \frac{\omega L}{c} = \frac{2}{\theta} \frac{Lv}{r_h^2 c}, \quad (8)$$

which is indeed independent of frequency if θ is. Moreover, if θ has the functional form

$$\theta = \frac{\theta_{scale}}{1 + \theta_{vel}|u_{pore}|}, \quad (9)$$

then LC is linearly dependent on velocity, as observed.

Taking this prescription, the best fits shown in Fig. 3 are

$$\theta^{-1} = 1.25 + 0.0048 \text{ Re}, \quad (10)$$

for the coarse mesh,

$$\theta^{-1} = 2.08 + 0.0237 \text{ Re}, \quad (11)$$

for the fine mesh.

The original formulations of (13) and (14) were based on the assumption that the velocity through the holes of the mesh would be an important correlating parameter, and hence the losses should depend on the 2-D projected porosity. Whichever porosity is selected, the coefficients in these correlations are very different for the two meshes, so no universal prescription can be proposed.

As seen in Fig. 3, very satisfactory, correlations have been established to describe the viscous characteristics of the two meshes in the first series of tests over a wide frequency range in a form that is useful in thermoacoustic calculations, to a precision of $\pm 10\%$. However, the results in the second series of tests indicate that much of this scatter can be eliminated by acknowledging that θ is actually frequency-dependent. Moreover, the difference in the fine mesh results from the two tests suggests that the tightness of packing is also a factor.

6. Conclusion

Acoustic pressure losses have been measured for two grades of randomly-orientated stainless steel, woven mesh (34# and 75#) in two different configurations over the frequency range 28–112 Hz. The change in impedance in all four cases has been shown to have a component that is linearly-related to the acoustic velocity. Three of the data sets converge at low amplitude and low frequency within the expected accuracy of the measurements to the values derived by Swift and Ward [2] from the steady-flow correlations of Kays and London [3], based on the data of Tong and London [4]. However, the fourth set (for the 75# mesh) have 20–30% lower pressure loss than predicted by Swift and Ward [2], possibly due to poor compaction of the meshes.

For the results from the second configuration, a correlation of the departures from the steady-flow values with frequency (made dimensionless by the viscous penetration time) was attempted. The individual data sets both increased monotonically with frequency. Geometrical differences may have been responsible for causing these coarse mesh data to lie some 25% below where they were expected from the fine mesh results.

Taking the formulation of the momentum equation developed extensively by Swift [1], the independence of frequency in the loss data has been shown in Appendix A to be consistent with the solutions for a variety of straight-channel types of regenerator in the low-frequency limit of large viscous penetration depth. This leads to a simple empirical correlation of the data for pressure loss, without invoking friction factor correlations, although they are equivalent. It is also shown that the acoustic pressure losses are much greater than the acoustic velocity reductions due to thermal dissipation, on a percentage basis.

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Appendix A. Viscous and thermal losses in acoustics

In Rott's acoustic approximation [10], as developed by Swift [1], the momentum equation can be written

$$\rho \frac{\partial u'}{\partial t} = -\frac{dp_a}{dx} + \mu \left(\frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial z^2} \right), \quad (A1)$$

with $u' = 0$ at a solid surface. This can be transformed into

$$\rho \frac{\partial u'}{\partial t} = -[1 - h_v(y, z)] \frac{dp_a}{dx}, \quad (A2)$$

and then the unknown function $h_v(y, z)$ satisfying

$$\frac{\partial^2 h_v}{\partial y^2} + \frac{\partial^2 h_v}{\partial z^2} = 2i \frac{h_v}{\delta_v^2}, \quad (A3)$$

integrated with respect to y and z to obtain

$$\frac{dp_a}{dx} = -\frac{i\omega\rho}{(1-f_v)}u, \quad (A4)$$

where p_a and u are the one-dimensional parameters, averaged over the transverse directions, and f_v is obtained by averaging h_v in those directions. Thus the viscous pressure loss across an obstruction can be written

$$\frac{\Delta p_a}{\rho c u} \equiv \Delta \zeta(\omega) = -\frac{i\omega \Delta x}{c(1-f_v)}. \quad (A5)$$

Similarly the energy equation in the absence of an axial temperature gradient,

$$\rho C_p \frac{\partial T'_a}{\partial t} = \frac{dp_a}{dt} + k \left(\frac{\partial^2 T'_a}{\partial y^2} + \frac{\partial^2 T'_a}{\partial z^2} \right), \quad (A6)$$

can be transformed into

$$\rho C_p T'_a = [1 - h_k(y, z)] p_a, \quad (A7)$$

where

$$\frac{\partial^2 h_k}{\partial y^2} + \frac{\partial^2 h_k}{\partial z^2} = 2i \frac{h_k}{\delta_k^2}, \quad (A8)$$

with $T'_a = 0$ at an isothermal surface. Then, invoking continuity,

$$i\omega\rho_a + \frac{d}{dx}(\rho u) = 0, \quad (A9)$$

and the perfect gas relationship

$$\frac{p_a}{P} = \frac{\rho_a}{\rho} + \frac{T_a}{T}, \quad (A10)$$

$$\frac{du}{dx} = -\frac{i\omega}{\gamma P} [1 + (\gamma - 1)f_k] p_a, \quad (A11)$$

so that the velocity change due to thermal relaxation can be written

$$\rho c \Delta u = -\frac{i\omega \Delta x}{c} [1 + (\gamma - 1)f_k] p_a. \quad (A12)$$

Thus the total acoustic dissipation associated with the obstruction is

$$\begin{aligned} -\frac{A}{2} \operatorname{Re}[\Delta(p_a u)] &= -\frac{A}{2} \operatorname{Re}[p_a \Delta u + \Delta p_a u] \\ &= \frac{A\omega \Delta x}{2c} \operatorname{Im} \left[\{1 + (\gamma - 1)f_k\} \frac{|p_a|^2}{\rho c} + \frac{\rho c |u|^2}{(1-f_v)} \right]. \end{aligned} \quad (A13)$$

Swift [1] shows that the functions f are very similar for a variety of straight channels when $r_h/\delta_{v,k} < 0.8$, where $\delta_v = \sqrt{\frac{2\nu}{\omega}}$ and $\delta_k = \sqrt{\frac{2\kappa}{\omega}}$ are the viscous and thermal penetration depths, respectively. For parallel plates, the accurate expression is

$$f = \frac{\tanh \left\{ (1+i) \frac{y_0}{\delta} \right\}}{(1+i) \frac{y_0}{\delta}}, \quad (A14)$$

where y_0 is the channel half-width. When expanded, it can be shown that in the limit $r_h/\delta \equiv y_0/\delta \ll 1$

$$f \rightarrow 1 - \frac{2i}{3} \left(\frac{y_0}{\delta} \right)^2 - \frac{8}{15} \left(\frac{y_0}{\delta} \right)^4. \quad (A15)$$

We may postulate that a reasonable second-order representation of f for the tortuous paths represented by randomly-oriented meshes is

$$f_v = 1 - i \left(\frac{r_h}{\delta_v} \right)^2 \theta, \quad (A16)$$

where θ is an empirical function of the Reynolds number for each type of mesh. Thus, defining the non-dimensional pressure loss in Eq. (A5) as LC ,

$$LC = \frac{1}{\theta} \left(\frac{\delta_v}{r_h} \right)^2 \frac{\omega L}{c} = \frac{2Lv}{\theta r_h^2 c}, \quad (A17)$$

which could be independent of frequency if θ is.

Moreover, since f_k is obtained by an integration of the solution to the same equation with the same boundary condition as f_v , apart from the change in δ , we may take

$$f_k = 1 - i \left(\frac{r_h}{\delta_k} \right)^2 \theta, \quad (A18)$$

with the same parameter θ . The ratio of the two contributions to the dissipation in Eq. (A13) is

$$\frac{\text{viscous dissipation}}{\text{thermal dissipation}} = \frac{(\rho c |u|)^2}{|p_a|^2} \frac{\delta_v^2 (1-\gamma)}{\delta_k^2} = \frac{1}{|Z|^2} \operatorname{Pr} (1-\gamma). \quad (A19)$$

Moreover,

$$\left(\frac{\Delta u}{u} \right) / \left(\frac{\Delta p_a}{p_a} \right) = \gamma \theta |Z|^2 \left(\frac{r_h}{\delta_v} \right)^2, \quad (A20)$$

and for the values measured in the first series, this ratio is <0.02 at 100 Hz and <0.01 at 40 Hz. Thus the pressure loss is far more significant than the change in velocity in these circumstances.

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